

# regression\_lineaire

March 16, 2019

## 1 Régression linéaire et résultats numériques

Ce notebook s'intéresse à la façon d'interpréter les résultats d'une régression linéaire lorsque les variables sont corrélées puis il explore une façon d'associer arbre de décision et régression linéaire pour construire une régression linéaire par morceaux.

```
In [1]: from jupyterhelper import add_notebook_menu
        add_notebook_menu()
```

```
Out[1]: <IPython.core.display.HTML object>
```

```
In [2]: %matplotlib inline
```

### 1.1 Un cas simple

Une façon d'interpréter des résultats statistiques est de les calculer dans un cas où la réponse cherchée est connue. On simule un modèle simple  $Y = \alpha X_1 + 0.X_2 + \epsilon$  et on calcule une régression linéaire. On suppose que  $X_1, X_2, \epsilon$  sont des variables aléatoires gaussiennes de même variance et moyenne.

```
In [3]: import numpy.random as npr
        eps = npr.normal(1000)
        X = npr.normal(size=(1000, 3))
        alpha = 2
        Y = alpha * X[:,0] + X[:, 2]
        X.shape, Y.shape
```

```
Out[3]: ((1000, 3), (1000,))
```

```
In [4]: from numpy import corrcoef
        corrcoef(X.T)
```

```
Out[4]: array([[ 1.          , -0.0312982 ,  0.05188551],
               [-0.0312982 ,  1.          , -0.00356494],
               [ 0.05188551, -0.00356494,  1.          ]])
```

```
In [5]: from statsmodels.regression.linear_model import OLS
```

```
In [6]: model = OLS(Y,X[:, :2])
        results = model.fit()
        su = results.summary()
        su
```

```
Out[6]: <class 'statsmodels.iolib.summary.Summary'>
"""
```

```

                        OLS Regression Results
=====
Dep. Variable:          y      R-squared:                0.815
Model:                 OLS    Adj. R-squared:           0.815
Method:                Least Squares  F-statistic:             2204.
Date:                  Mon, 15 Oct 2018  Prob (F-statistic):       0.00
Time:                  10:34:12  Log-Likelihood:          -1385.2
No. Observations:     1000      AIC:                     2774.
Df Residuals:          998      BIC:                     2784.
Df Model:              2
Covariance Type:      nonrobust
=====
                    coef    std err          t      P>|t|      [0.025    0.975]
-----
x1                 2.0519     0.031     66.347     0.000     1.991     2.113
x2                -0.0032     0.033     -0.097     0.922    -0.067     0.061
=====
Omnibus:                 0.709   Durbin-Watson:           1.990
Prob(Omnibus):           0.701   Jarque-Bera (JB):         0.674
Skew:                    0.063   Prob(JB):                 0.714
Kurtosis:                3.010   Cond. No.                  1.07
=====
```

```
Warnings:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
"""
```

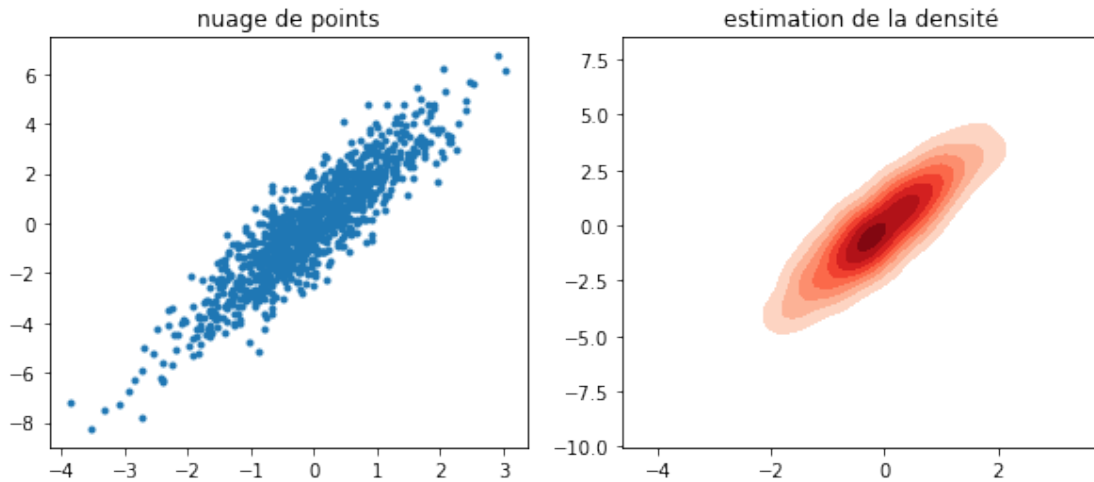
```
In [7]: results.rsquared, results.rsquared_adj
```

```
Out[7]: (0.8153831029946165, 0.8150131292531227)
```

On vérifie que le coefficient devant  $X_1$  est non nul (P-value nulle, 0 n'est pas l'intervalle de confiance). Le coefficient devant  $X_2$  n'est pas nul mais presque, la P-value est élevée, le coefficient  $R^2$  est élevé. Dessinons.

```
In [8]: import matplotlib.pyplot as plt
import seaborn
fig, ax = plt.subplots(1, 2, figsize=(10,4))
ax[0].plot(X[:, 0], Y, '.')
seaborn.kdeplot(X[:, 0], Y, cmap="Reds", shade=True, shade_lowest=False, ax=ax[1])
ax[0].set_title("nuage de points")
ax[1].set_title("estimation de la densité");
```

```
c:\python370_x64\lib\site-packages\scipy\stats\stats.py:1713: FutureWarning: Using a non-tuple sequence
return np.add.reduce(sorted[indexer] * weights, axis=axis) / sumval
```

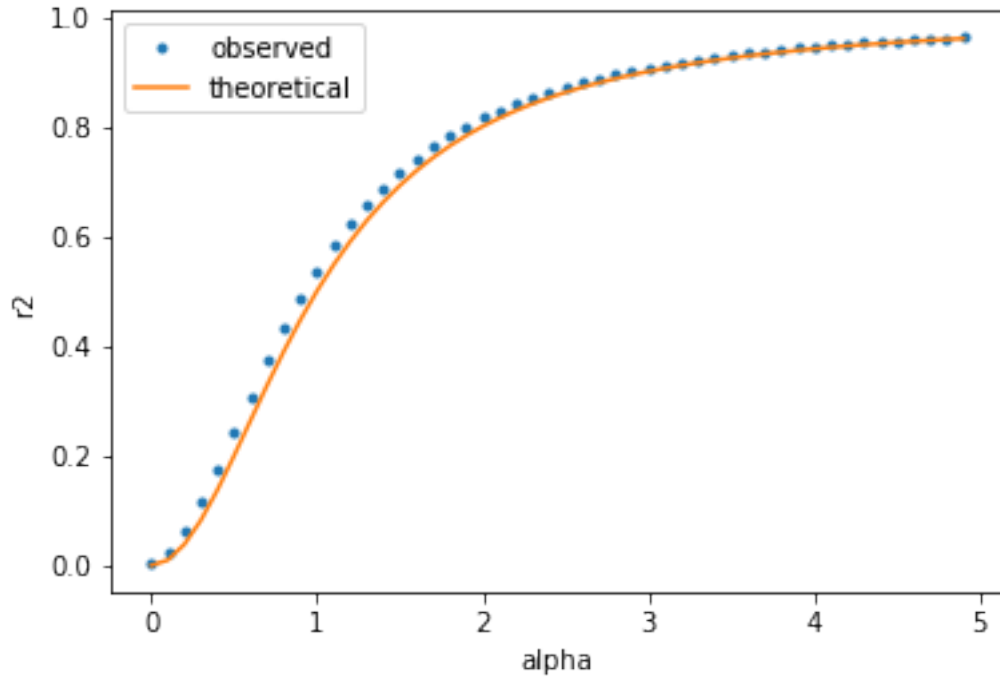


## 1.2 Evolution de R2

Dans la régression précédente, le coefficient  $R^2$  transcrit en quelque sorte la part du bruit  $\epsilon$  par rapport au terme  $\alpha X_1$ . Faisons varier  $\alpha$ .

```
In [9]: alphas = []
        r2s = []
        for a in [0.1 * i for i in range(0, 50)]:
            Y = a*X[:,0] + X[:, 2]
            model = OLS(Y,X[:, :2])
            results = model.fit()
            alphas.append(a)
            r2s.append(results.rsquared)
```

```
In [10]: fig, ax = plt.subplots(1, 1)
         ax.plot(alphas, r2s, '.', label="observed")
         ax.plot(alphas, [a**2/(1+a**2) for a in alphas], label='theoretical')
         ax.set_xlabel("alpha")
         ax.set_ylabel("r2")
         ax.legend();
```



Dans ce cas de régression simple, la valeur à prédire est  $y_i$ , la valeur prédite est  $\hat{y}_i = \alpha X_{1i}$  et la moyenne  $\bar{y} = \alpha \bar{X}_1 + \bar{\epsilon} = 0$ .

$$R^2 = 1 - \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} = 1 - \frac{\mathbb{V}\epsilon}{\alpha^2 \mathbb{V}X_1 + \mathbb{V}\epsilon} = 1 - \frac{1}{1 + \alpha^2} = \frac{\alpha^2}{1 + \alpha^2}$$

### 1.3 Deux variables corrélées

On ne change pas le modèle mais on fait en sorte que  $X_2 = X_1$ . Les deux variables sont corrélées.

```
In [11]: X[:, 1] = X[:, 0]
         Y = 2*X[:,0] + X[:, 2]
         model = OLS(Y,X[:, :2])
         results = model.fit()
         results.summary()
```

```
Out[11]: <class 'statsmodels.iolib.summary.Summary'>
        """
```

```

                                OLS Regression Results
=====
Dep. Variable:                    y      R-squared:                    0.810
Model:                            OLS    Adj. R-squared:                0.810
Method:                            Least Squares  F-statistic:                  4271.
Date:                               Mon, 27 Nov 2017  Prob (F-statistic):          0.00
Time:                               12:06:03      Log-Likelihood:              -1411.2
No. Observations:                  1000      AIC:                         2824.
Df Residuals:                      999      BIC:                         2829.
Df Model:                          1
Covariance Type:                  nonrobust
```

	coef	std err	t	P> t	[0.025	0.975]
x1	1.0288	0.016	65.349	0.000	0.998	1.060
x2	1.0288	0.016	65.349	0.000	0.998	1.060
Omnibus:		8.165	Durbin-Watson:			1.944
Prob(Omnibus):		0.017	Jarque-Bera (JB):			6.024
Skew:		-0.064	Prob(JB):			0.0492
Kurtosis:		2.642	Cond. No.			1.61e+16

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The smallest eigenvalue is 7.69e-30. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

In [12]: model.rank

Out[12]: 1

Les variables corrélées n'ont pas l'air de déranger l'algorithme de résolution car il utilise la méthode SVD pour résoudre le même problème dans un espace de moindre dimension. Le problème survient que les deux variables ne sont pas complètement corrélées. On étudie le modèle  $Y \sim X_1 + X_2'$  avec  $X_2' = \alpha X_1 + (1 - \alpha)X_2$  et on réduit la variance du bruit pour en diminuer les effets.

In [13]: X\_ = npr.normal(size=(1000, 3))

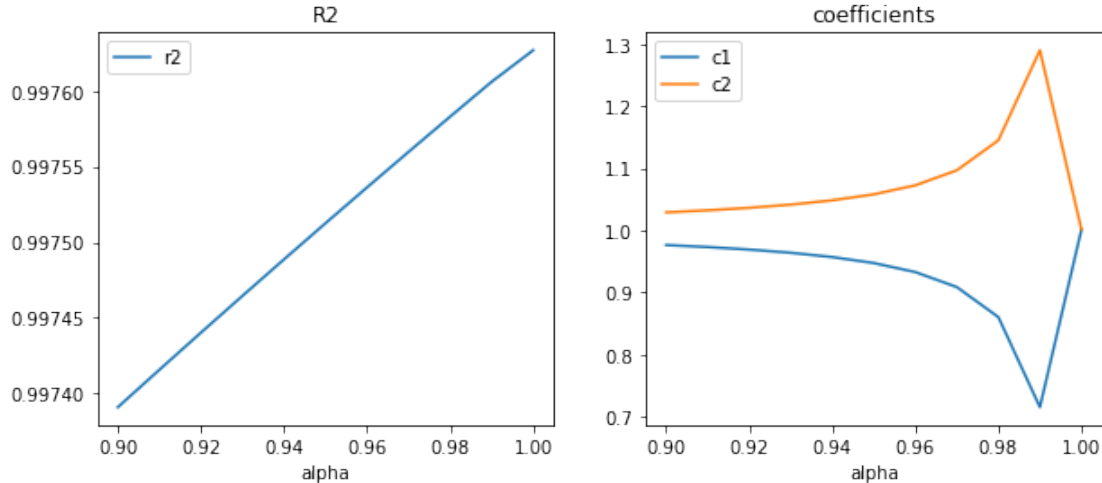
```
In [14]: alphas = [0.9 + i * 0.01 for i in range(0,11)]
res = []
for a in alphas:
    X = X_.copy()
    X[:, 1] = a * X[:, 0] + (1-a) * X[:, 1]
    Y = X[:, 0] + X[:, 1] + 0.1 * X[:, 2]
    model = OLS(Y,X[:, :2])
    results = model.fit()
    res.append(dict(alpha=a, r2=results.rsquared, rank=model.rank, c1=results.params[0], c2=results.params[1]))

import pandas
df = pandas.DataFrame(res)
df = df.set_index('alpha')
df
```

```
Out[14]:
```

alpha	c1	c2	r2	rank
0.90	0.976370	1.028982	0.997391	2
0.91	0.973150	1.032202	0.997416	2
0.92	0.969125	1.036227	0.997440	2
0.93	0.963950	1.041402	0.997464	2
0.94	0.957049	1.048303	0.997489	2
0.95	0.947389	1.057963	0.997513	2
0.96	0.932898	1.072454	0.997536	2
0.97	0.908747	1.096605	0.997560	2
0.98	0.860444	1.144908	0.997583	2
0.99	0.715536	1.289816	0.997606	2
1.00	1.001279	1.001279	0.997627	1

```
In [15]: fig, ax = plt.subplots(1,2, figsize=(10,4))
df[["r2"]].plot(ax=ax[0])
df[["c1", "c2"]].plot(ax=ax[1])
ax[0].set_title("R2")
ax[1].set_title("coefficients");
```



Le  $r^2$  augmente quand la corrélation augmente mais les coefficients sont moins fiables. Les résultats devraient être sensiblement identiques en théorie mais en pratique, plus le déterminant devient proche de zéro, plus l'ordinateur est limité par sa précision numérique. Pour en savoir plus, vous pouvez lire un examen écrit que j'ai rédigé, en python bien sûr : [Examen Programmation ENSAE première année 2006](#). Cette précision est aux alentours de  $10^{-15}$  ce qui correspond à la précision numérique des `double`.

```
In [16]: alphas = [1 - 10**(-i) for i in range(10,18)]
res = []
for a in alphas:
    X = X_.copy()
    X[:, 1] = a * X[:, 0] + (1-a) * X[:, 1]
    Y = X[:, 0] + X[:, 1] + X[:, 2]
    model = OLS(Y,X[:, :2])
    results = model.fit()
    res.append(dict(alpha_1=a-1, r2=results.rsquared, rank=model.rank, c1=results.params[0], c2=results.params[1]))

import pandas
df = pandas.DataFrame(res)
df = df.set_index('alpha_1')
df
```

```
Out[16]:
```

	c1	c2	r2	rank
alpha_1				
-1.000000e-10	-2.898180e+08	2.898180e+08	0.811519	2
-1.000000e-11	-2.898201e+09	2.898201e+09	0.811519	2
-9.999779e-13	-2.898941e+10	2.898941e+10	0.811519	2
-1.000311e-13	-2.891422e+11	2.891422e+11	0.811518	2
-9.992007e-15	-2.915101e+12	2.915101e+12	0.811508	2
-9.992007e-16	1.012789e+00	1.012789e+00	0.811359	2
-1.110223e-16	1.012789e+00	1.012789e+00	0.811359	1
0.000000e+00	1.012789e+00	1.012789e+00	0.811359	1

On fait un dernier test avec `scikit-learn` pour vérifier que l'algorithme de résolution donne des résultats similaires pour un cas où le déterminant est quasi-nul.

```
In [17]: from sklearn.linear_model import LinearRegression
         from sklearn.metrics import r2_score

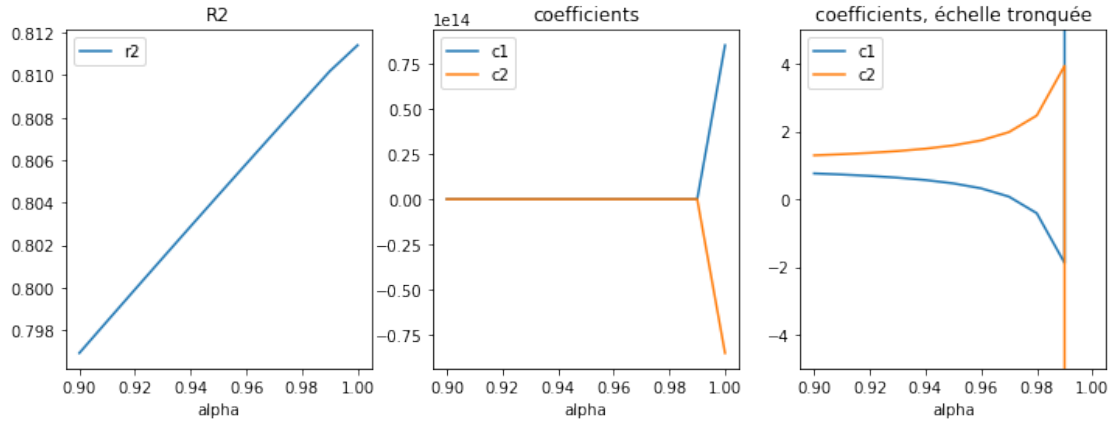
         alphas = [0.9 + i * 0.01 for i in range(0,11)]
         res = []
         for a in alphas:
             X = X_.copy()
             X[:, 1] = a * X[:, 0] + (1-a) * X[:, 1]
             Y = X[:, 0] + X[:, 1] + X[:, 2]
             model = LinearRegression()
             model.fit(X[:, :2], Y)
             r2 = r2_score(Y, model.predict(X[:, :2]))
             res.append(dict(alpha=a, c1=model.coef_[0], c2=model.coef_[1], r2=r2))

         import pandas
         df = pandas.DataFrame(res)
         df = df.set_index('alpha')
         df
```

```
Out[17]:
```

	c1	c2	r2
alpha			
0.90	7.601931e-01	1.293903e+00	0.796916
0.91	7.275372e-01	1.326559e+00	0.798417
0.92	6.867173e-01	1.367379e+00	0.799911
0.93	6.342346e-01	1.419862e+00	0.801399
0.94	5.642576e-01	1.489839e+00	0.802880
0.95	4.662898e-01	1.587807e+00	0.804355
0.96	3.193382e-01	1.734758e+00	0.805823
0.97	7.441878e-02	1.979678e+00	0.807283
0.98	-4.154200e-01	2.469516e+00	0.808736
0.99	-1.884936e+00	3.939033e+00	0.810182
1.00	8.512221e+13	-8.512221e+13	0.811404

```
In [18]: fig, ax = plt.subplots(1,3, figsize=(12,4))
         df[["c1", "c2"]].plot(ax=ax[1])
         df[["c1", "c2"]].plot(ax=ax[2])
         df[["r2"]].plot(ax=ax[0])
         ax[0].set_title("R2")
         ax[1].set_title("coefficients")
         ax[2].set_ylim([-5, 5])
         ax[2].set_title("coefficients, échelle tronquée");
```



Le second graphe est trompeur mais il ne faut pas oublier de regarder l'échelle de l'axe des ordonnées.

## 1.4 Indicatrices

$X_1$  est une variable aléatoire gaussienne. On teste maintenant un modèle  $Y = X'_1 + X'_2 + \epsilon$  avec  $X'_1 = X_1 \mathbf{1}_{X_1 < 0}$  et  $X'_2 = X_1 \mathbf{1}_{X_1 \geq 0}$ .

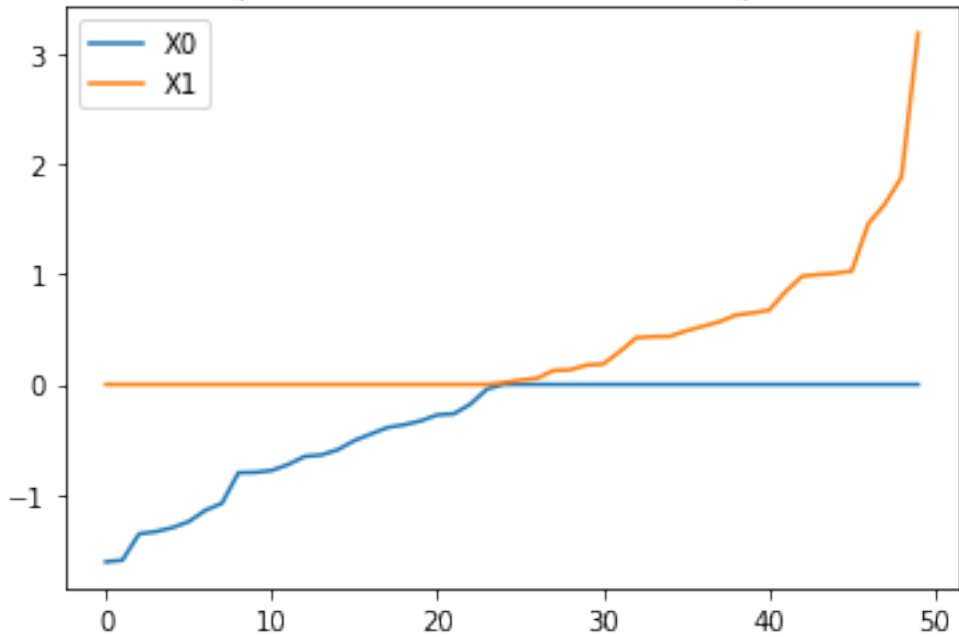
```
In [19]: X = npr.normal(size=(1000, 3))
X[:, 1] = X[:, 0]
X[X[:, 0] >= 0, 0] = 0
X[X[:, 1] < 0, 1] = 0
Y = X[:, 0] + X[:, 1] + X[:, 2]
corrcoef(X.T)
```

```
Out[19]: array([[ 1.          ,  0.47358312, -0.03083914],
 [ 0.47358312,  1.          , -0.01293737],
 [-0.03083914, -0.01293737,  1.          ]])
```

```
In [20]: from pandas import DataFrame
names = ["X%d" % i for i in range(X.shape[1]-1)]
ax = DataFrame(X[:50,:2], columns=names).sort_values(names).reset_index(drop=True).plot()
ax.set_title("Représentation des features tronquées");
```



Représentation des features tronquées



```
In [21]: model = OLS(Y,X[:, :3])
         results = model.fit()
         results.summary()
```

```
Out[21]: <class 'statsmodels.iolib.summary.Summary'>
        """
```

```

                                OLS Regression Results
=====
Dep. Variable:                  y      R-squared:                  1.000
Model:                          OLS    Adj. R-squared:             1.000
Method:                        Least Squares  F-statistic:                2.212e+33
Date:                            Mon, 15 Oct 2018  Prob (F-statistic):          0.00
Time:                             10:56:29    Log-Likelihood:             33713.
No. Observations:                1000      AIC:                       -6.742e+04
Df Residuals:                     997      BIC:                       -6.740e+04
Df Model:                          3
Covariance Type:                  nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
x1	1.0000	2.42e-17	4.14e+16	0.000	1.000	1.000
x2	1.0000	2.39e-17	4.18e+16	0.000	1.000	1.000
x3	1.0000	1.73e-17	5.78e+16	0.000	1.000	1.000

```

=====
Omnibus:                        4.249    Durbin-Watson:              2.031
Prob(Omnibus):                   0.119    Jarque-Bera (JB):           4.338
Skew:                             -0.107    Prob(JB):                   0.114
Kurtosis:                         3.242    Cond. No.                   1.40
=====

```

Warnings:

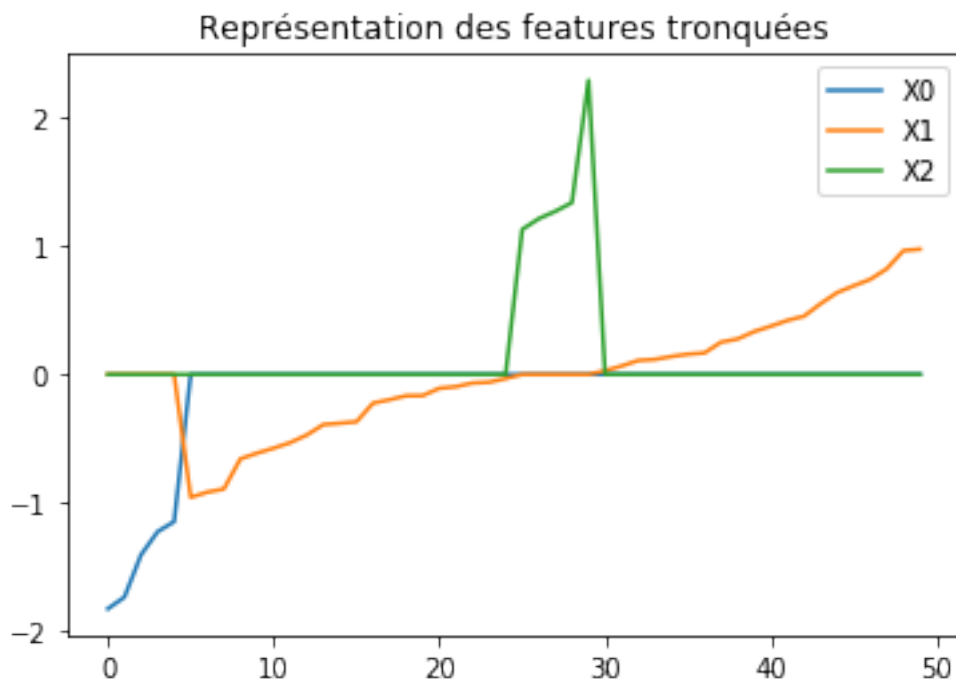
```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
"""
```

On découpe en trois.

```
In [22]: import numpy
X = npr.normal(size=(1000, 4))
for i in range(0, 3):
    X[:, i] = X[:, 0]
X[:, 3] = X[:, 2]
X[X[:, 0] > -1, 0] = 0
X[(X[:, 0] < -1) | (X[:, 0] > 1), 1] = 0
X[X[:, 0] < 1, 2] = 0
Y = X[:, 0] + X[:, 1] + X[:, 2] + X[:, 3]
corrcoef(X.T)
```

```
Out[22]: array([[ 1.          , -0.00347584,  0.16846101,  0.06722762],
                [-0.00347584,  1.          ,  0.00326437, -0.04707208],
                [ 0.16846101,  0.00326437,  1.          ,  0.08754832],
                [ 0.06722762, -0.04707208,  0.08754832,  1.          ]])
```

```
In [23]: from pandas import DataFrame
names = ["X%d" % i for i in range(X.shape[1]-1)]
ax = DataFrame(X[:50,:3], columns=names).sort_values(names).reset_index(drop=True).plot()
ax.set_title("Représentation des features tronquées");
```



```
In [24]: model = OLS(Y,X[:, :4])
results = model.fit()
results.summary()
```

```
Out [24]: <class 'statsmodels.iolib.summary.Summary'>
        """
```

```

                                OLS Regression Results
=====
Dep. Variable:                    y      R-squared:                    1.000
Model:                            OLS      Adj. R-squared:                1.000
Method:                            Least Squares      F-statistic:                    1.910e+32
Date:                            Mon, 15 Oct 2018      Prob (F-statistic):              0.00
Time:                            10:57:27      Log-Likelihood:                  32608.
No. Observations:                  1000      AIC:                            -6.521e+04
Df Residuals:                      996      BIC:                            -6.519e+04
Df Model:                          4
Covariance Type:                  nonrobust
=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
x1              1.0000      8.75e-17      1.14e+16      0.000          1.000          1.000
x2              1.0000      1.22e-16      8.23e+15      0.000          1.000          1.000
x3              1.0000      8.33e-17      1.2e+16      0.000          1.000          1.000
x4              1.0000      5.23e-17      1.91e+16      0.000          1.000          1.000
=====
Omnibus:                            457.967      Durbin-Watson:                1.816
Prob(Omnibus):                       0.000      Jarque-Bera (JB):              1967.636
Skew:                                 -2.198      Prob(JB):                      0.00
Kurtosis:                             8.282      Cond. No.                      2.35
=====
```

```
Warnings:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
    """
```

## 1.5 Régression linéaire par morceaux

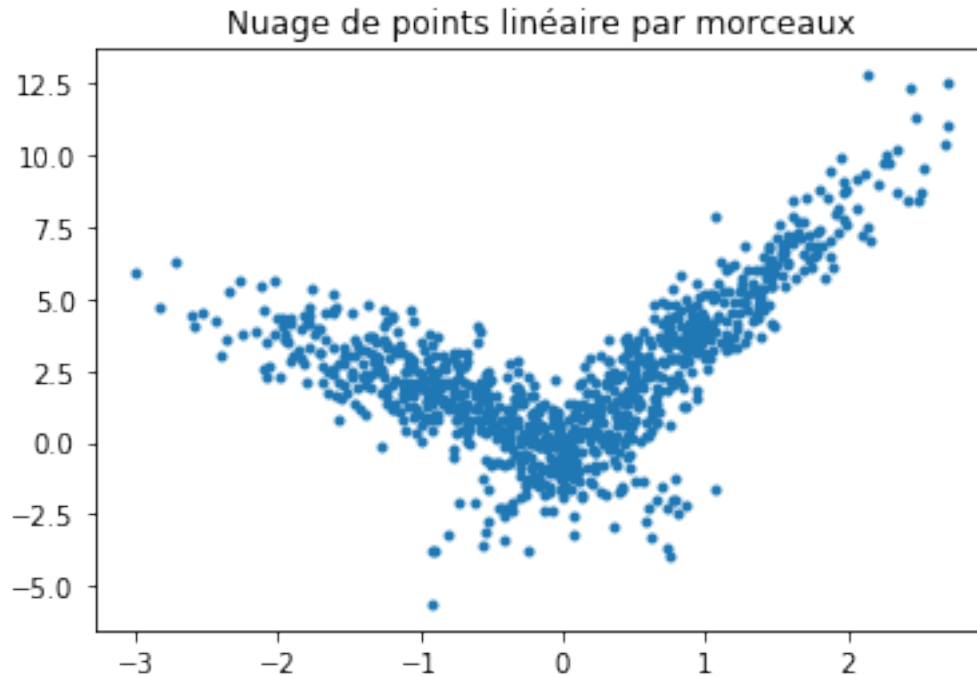
On se place dans un cas particulier où le problème est linéaire par morceaux :

$$Y = -2X_1 \mathbf{1}_{X_1 + \epsilon_1 < 0} + 4X_1 \mathbf{1}_{X_1 + \epsilon_1 > 0} + \epsilon_2$$

La régression donne de très mauvais résultat sur ce type de problèmes mais on cherche une façon systématique de découper le problème en segments linéaires.

```
In [25]: X = npr.normal(size=(1000,4))
        alpha = [4, -2]
        t = (X[:, 0] + X[:, 3] * 0.5) > 0
        switch = numpy.zeros(X.shape[0])
        switch[t] = 1
        Y = alpha[0] * X[:, 0] * t + alpha[1] * X[:, 0] * (1-t) + X[:, 2]
```

```
In [26]: fig, ax = plt.subplots(1, 1)
        ax.plot(X[:, 0], Y, ".")
        ax.set_title("Nuage de points linéaire par morceaux");
```



```
In [27]: model = OLS(Y,X[:, :1])
         results = model.fit()
         results.summary()
```

```
Out[27]: <class 'statsmodels.iolib.summary.Summary'>
        """
```

```

                                OLS Regression Results
=====
Dep. Variable:                  y    R-squared:                0.094
Model:                          OLS  Adj. R-squared:           0.093
Method:                         Least Squares  F-statistic:             104.0
Date:                            Mon, 15 Oct 2018  Prob (F-statistic):      2.69e-23
Time:                             10:59:28    Log-Likelihood:         -2594.9
No. Observations:                1000    AIC:                    5192.
Df Residuals:                    999      BIC:                    5197.
Df Model:                         1
Covariance Type:                 nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
x1	1.0252	0.101	10.197	0.000	0.828	1.222

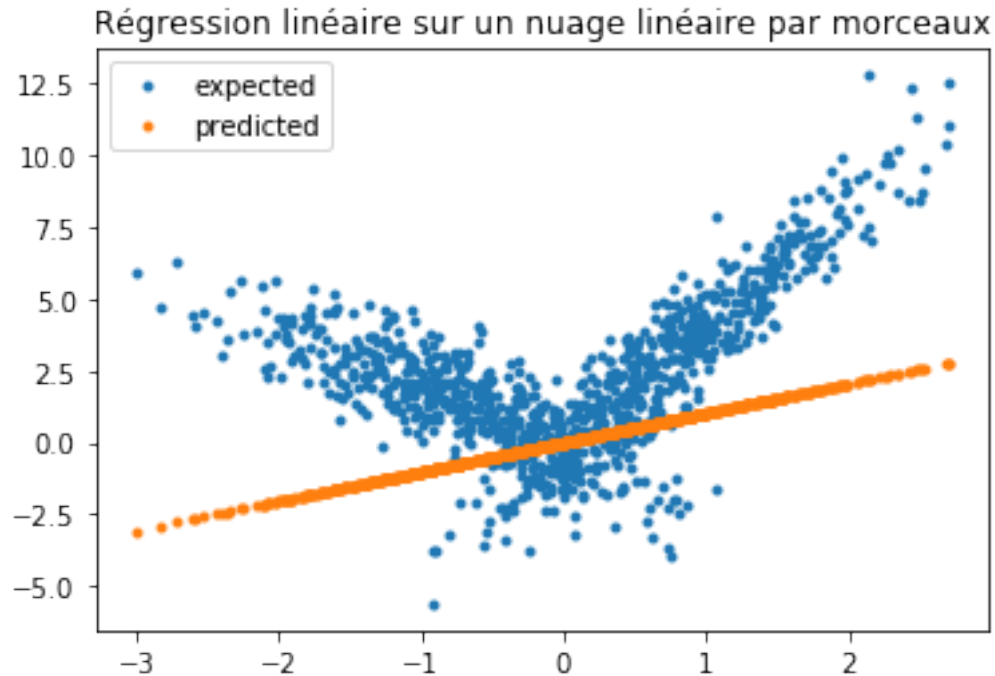
```
=====
Omnibus:                        3.882    Durbin-Watson:           1.092
Prob(Omnibus):                  0.144    Jarque-Bera (JB):       3.834
Skew:                          0.151    Prob(JB):                0.147
Kurtosis:                      3.015    Cond. No.                1.00
=====
```

```
Warnings:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
"""
```

```
In [28]: yp = results.predict(X[:, :1])
```

```
In [29]: fig, ax = plt.subplots(1, 1)
ax.plot(X[:, 0], Y, ".", label="expected")
ax.plot(X[:, 0], yp, ".", label="predicted")
ax.legend()
ax.set_title("Régression linéaire sur un nuage linéaire par morceaux");
```

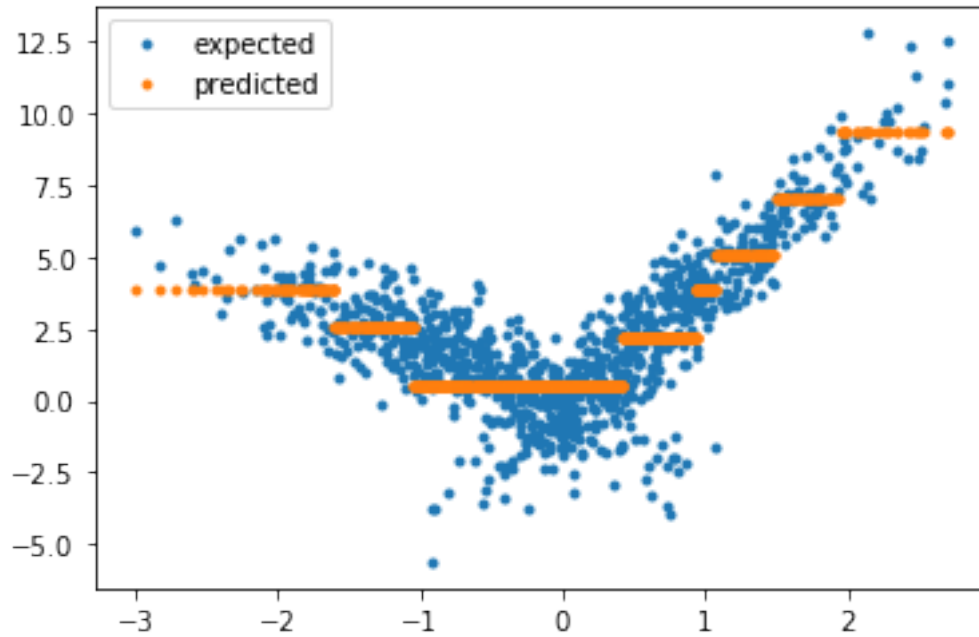


Passons à un arbre de décision qui n'est pas le meilleur modèle mais on va détourner ses résultats pour revenir à un problème de régression par morceaux.

```
In [30]: from sklearn.tree import DecisionTreeRegressor
model = DecisionTreeRegressor(min_samples_leaf=10, max_depth=3)
model.fit(X[:, :1], Y)
yp = model.predict(X[:, :1])
```

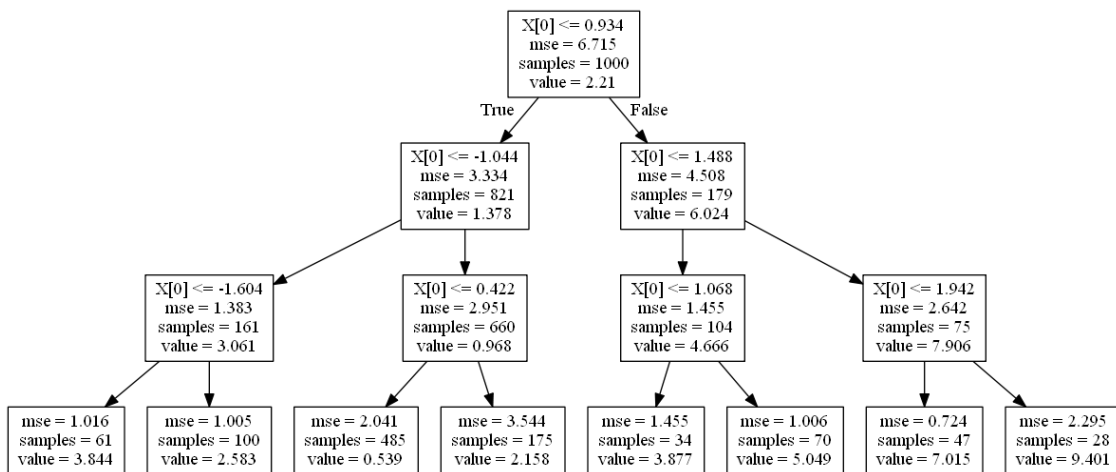
```
In [31]: fig, ax = plt.subplots(1, 1)
ax.plot(X[:, 0], Y, ".", label="expected")
ax.plot(X[:, 0], yp, ".", label="predicted")
ax.legend()
r2 = r2_score(Y, model.predict(X[:, :1]))
ax.set_title("Arbre de décision sur un nuage linéaire par morceaux\nR2=%f" % r2);
```

Arbre de décision sur un nuage linéaire par morceaux  
R2=0.703533



```
In [32]: from sklearn.tree import export_graphviz
export_graphviz(model, out_file="arbre.dot")
from pyensae.graphhelper import run_dot
run_dot("arbre.dot", "arbre.png")
from IPython.display import Image
Image("arbre.png")
```

Out [32]:



On extrait tous les seuils de l'arbre et on ajoute les milieux de segments.

```
In [33]: th = list(sorted(set(model.tree_.threshold)))
th += [(th[i] + th[i-1])/2 for i in range(1,len(th))]
th = list(sorted(th))
th
```

```
Out[33]: [-2.0,
-1.8018612563610077,
-1.6037225127220154,
-1.323736995458603,
-1.0437514781951904,
-0.3109976723790169,
0.4217561334371567,
0.678125374019146,
0.9344946146011353,
1.0011553764343262,
1.067816138267517,
1.2776717841625214,
1.4875274300575256,
1.7147845923900604,
1.9420417547225952]
```

On fait une régression sur les variables  $W_{i>0} = X_1 \mathbf{1}_{X_1 > t_i}$ ,  $W_0 = X_1$  où les  $(t_i)$  sont les seuils.

```
In [34]: W = numpy.zeros((X.shape[0], len(th)+1))
x = X[:, 0]
W[:, 0] = x
for i in range(len(th)):
    W[x > th[i], i+1] = x[x > th[i]]
```

```
In [35]: model = OLS(Y,W)
results = model.fit()
results.summary()
```

```
Out[35]: <class 'statsmodels.iolib.summary.Summary'>
"""
```

OLS Regression Results						
=====						
Dep. Variable:	y	R-squared:	0.849			
Model:	OLS	Adj. R-squared:	0.847			
Method:	Least Squares	F-statistic:	346.9			
Date:	Mon, 15 Oct 2018	Prob (F-statistic):	0.00			
Time:	11:07:03	Log-Likelihood:	-1697.7			
No. Observations:	1000	AIC:	3427.			
Df Residuals:	984	BIC:	3506.			
Df Model:	16					
Covariance Type:	nonrobust					
=====						
	coef	std err	t	P> t	[0.025	0.975]
-----						
x1	-1.8316	0.124	-14.806	0.000	-2.074	-1.589
x2	-0.0573	0.210	-0.273	0.785	-0.469	0.354
x3	-0.2440	0.235	-1.038	0.300	-0.705	0.217
x4	0.3291	0.218	1.508	0.132	-0.099	0.757
x5	-0.3610	0.206	-1.756	0.079	-0.764	0.042
x6	0.5528	0.197	2.811	0.005	0.167	0.939

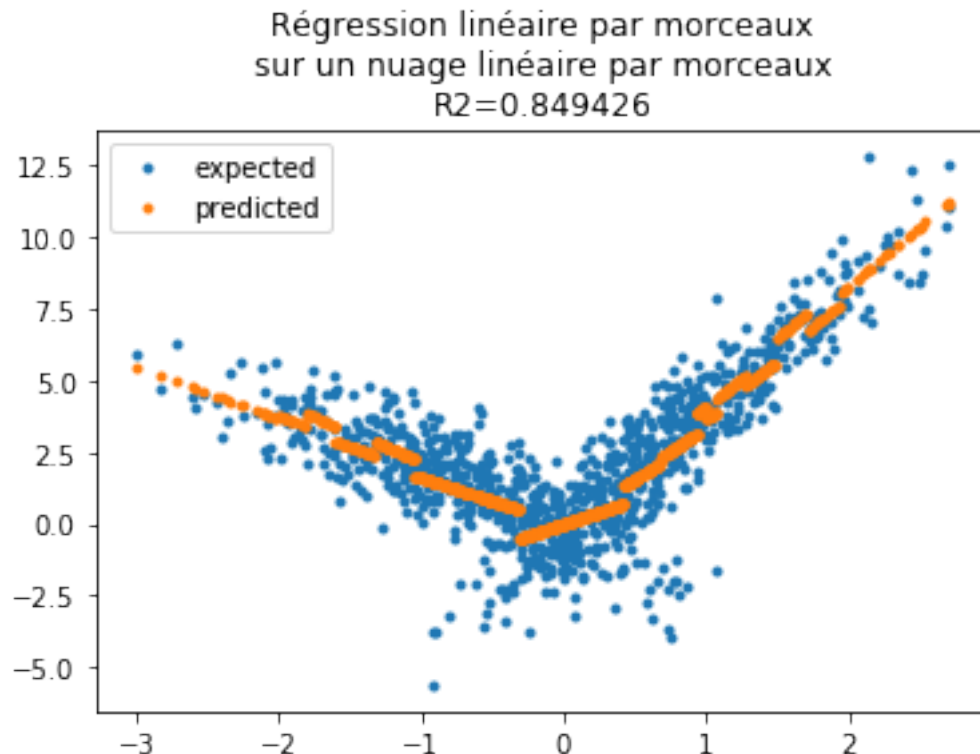
x7	3.2628	0.395	8.253	0.000	2.487	4.039
x8	1.4566	0.449	3.244	0.001	0.576	2.338
x9	0.2701	0.311	0.869	0.385	-0.340	0.880
x10	0.7213	0.374	1.928	0.054	-0.013	1.456
x11	-0.4599	0.457	-1.006	0.315	-1.357	0.437
x12	0.4177	0.378	1.105	0.269	-0.324	1.159
x13	-0.2703	0.253	-1.069	0.285	-0.766	0.226
x14	0.5325	0.226	2.360	0.018	0.090	0.975
x15	-0.3703	0.229	-1.618	0.106	-0.819	0.079
x16	0.1996	0.194	1.026	0.305	-0.182	0.581
=====						
Omnibus:		228.022	Durbin-Watson:			1.978
Prob(Omnibus):		0.000	Jarque-Bera (JB):			716.200
Skew:		-1.110	Prob(JB):			3.01e-156
Kurtosis:		6.502	Cond. No.			36.3
=====						

Warnings:

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
"""
```

Dessignons les résultats de la prédictions.

```
In [36]: yp = results.predict(W)
fig, ax = plt.subplots(1, 1)
ax.plot(X[:, 0], Y, ".", label="expected")
ax.plot(X[:, 0], yp, ".", label="predicted")
ax.legend()
ax.set_title("Régression linéaire par morceaux\nsur un nuage linéaire par morceaux\nR2=%f" % r
```





Le modèle nous suggère de ne garder que quelques seuils. En s'appuyant sur les p-values :

```
In [37]: keep = numpy.arange(len(results.pvalues))[results.pvalues < 0.05]
        keep
```

```
Out[37]: array([ 0,  5,  6,  7, 13])
```

```
In [38]: W2 = W[:, keep]
```

```
In [39]: model = OLS(Y,W2)
        results = model.fit()
        results.summary()
```

```
Out[39]: <class 'statsmodels.iolib.summary.Summary'>
        """
```

```

                        OLS Regression Results
=====
Dep. Variable:          y      R-squared:                0.846
Model:                  OLS    Adj. R-squared:         0.845
Method:                 Least Squares  F-statistic:           1094.
Date:                   Mon, 15 Oct 2018  Prob (F-statistic):    0.00
Time:                   11:07:38    Log-Likelihood:        -1708.6
No. Observations:      1000    AIC:                   3427.
Df Residuals:          995      BIC:                   3452.
Df Model:               5
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
x1	-1.9504	0.066	-29.574	0.000	-2.080	-1.821
x2	0.3384	0.148	2.287	0.022	0.048	0.629
x3	3.2628	0.397	8.209	0.000	2.483	4.043
x4	2.0247	0.385	5.260	0.000	1.269	2.780
x5	0.4635	0.119	3.901	0.000	0.230	0.697

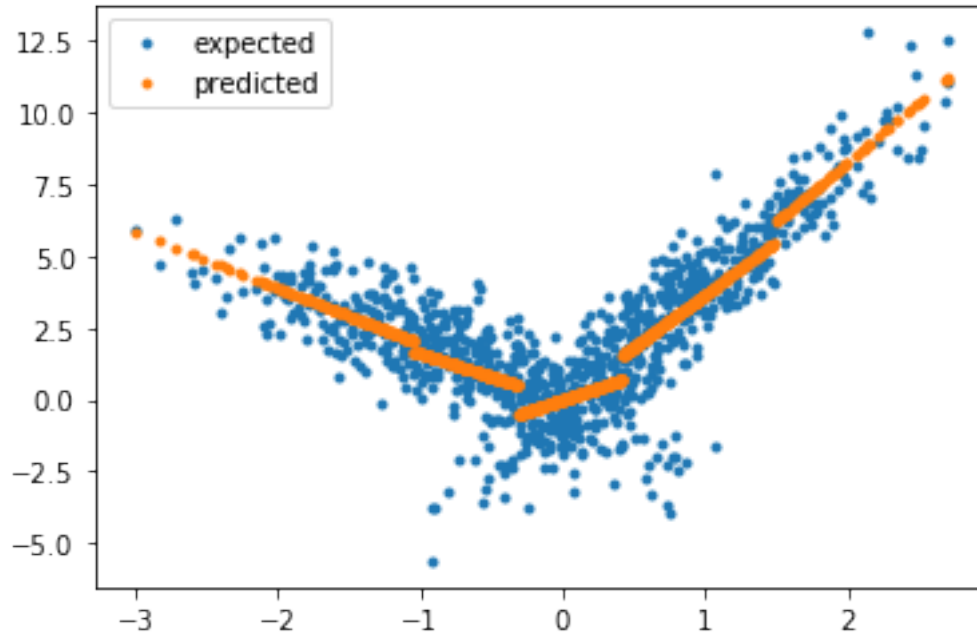
```
=====
Omnibus:                248.807    Durbin-Watson:         1.984
Prob(Omnibus):          0.000     Jarque-Bera (JB):      829.417
Skew:                   -1.190     Prob(JB):               7.84e-181
Kurtosis:                6.774     Cond. No.               20.1
=====
```

Warnings:

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
"""
```

```
In [40]: yp = results.predict(W2)
        fig, ax = plt.subplots(1, 1)
        ax.plot(X[:, 0], Y, ".", label="expected")
        ax.plot(X[:, 0], yp, ".", label="predicted")
        ax.legend()
        ax.set_title("Régression linéaire par morceaux\nsur un nuage linéaire par morceaux\n" +
                    "réduction du nombre de segments\nR2=%f" % results.rsquared);
```

Régression linéaire par morceaux  
sur un nuage linéaire par morceaux  
réduction du nombre de segments  
 $R^2=0.846099$



Le coefficient  $R^2$  est quasiment identique pour un nombre de segments moindre. Je me suis amusé à rendre ce code plus générique pour comparer la première étape, le découpage en morceaux, via deux modèles, un arbre de décision et le nouvel objet [KBinsDiscretizer](#) qui segmente une variable sans tenir compte de la cible. La régression n'est plus nécessaire linéaire : [Piecewise linear regression](#). Je me suis également amusé à faire de même pour une classification par morceaux [PiecewiseClassifier](#). Celle-ci pose quelques soucis pratiques car toutes les classes ne sont pas forcément représentées dans chaque compartiment...

In [41]: